

Magnetotransport in the insulating regime of Mn-doped GaAs

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We consider transport in the insulating regime of GaMnAs. We calculate the resistance, magnetoresistance, and Hall effect, assuming that the Fermi energy is in the region of localized states above the valence-band mobility edge. Both hopping and activated band transport contributions are included. The anomalous Hall current from band states is very different from the hopping Hall current and has extrinsic (skew) and intrinsic (Luttinger) contributions. Comparison with experiment allows us to assess the degree to which band and hopping contribution determine each of the three transport coefficients in a particular temperature range. There are strong indications that the insulating state transport in GaMnAs is controlled primarily by extended state, band edge, transport rather than by variable range hopping, as reported in the literature.

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I. INTRODUCTION

Magnetically doped semiconductors constitute a very active field of research for good reasons. As magnets, they are expected to keep some of the useful properties of the host system. As doped semiconductors, they have their own usual applications now combined with new functionality. For example, with quantum well (QW) multilayers and quantum dot (QD) Stranski-Krastanov growth technology,¹ it is possible that spins are localized in small clusters as in InMnAs (Ref. 2) and in QD layers of MnGaAs.¹ This could be used in building small magnets, memories, and spintronic switches. Therefore, localized state transport, magnetism of cluster-localized states, and tunneling are very interesting and important phenomena. However, we must begin with a clear understanding of the bulk materials, whose electronic transport properties in the insulating regime we investigate here. We demonstrate that the observed properties cannot be explained solely by hopping in homogeneous media. For this purpose, we need to develop the hopping theory alongside the band theory and compare their predictions. The approach in this paper has a wide range of applications and is, we believe, useful to the wider magnetism and device communities. The theory will enable us to treat resistance, magnetoresistance (MR), thermopower, and normal and anomalous Hall effects in the high resistance regime of diluted magnetic semiconductors.

Mn-doped semiconductors³ are disordered alloys, so one cannot expect sharp band edges, and indeed, one should observe Anderson localized states above (for holes) the mobility edges. The long-range ferromagnetism is mediated by holes, which lower their energy each time they interact with a manganese spin of opposite spin. The mobile valence-band p holes interact with the localized moments via the antiferromagnetic (AFM) exchange coupling,

$$H_{pd} = \frac{J_{pd}}{2} \sum_{m \in \{N_S\}} \sum_{s,s'} c_{ms}^\dagger \boldsymbol{\sigma}_{ss'} \cdot \mathbf{S}_m c_{ms'}, \quad (1)$$

where J_{pd} is the hole-Mn coupling and $\boldsymbol{\sigma}_{ss'}$ is a vector containing Pauli's matrices, i.e., $[\sigma^x, \sigma^y, \sigma^z]$. The indices, s and

s' , indicate which terms of the 2×2 matrix we are considering. Finally, \mathbf{S}_m is the Mn spin operator at site m and the c_{ms}^\dagger and c_{ms} are the creation and annihilation operators for a carrier of spin s at site m . This Hamiltonian, applied to GaMnAs, has been discussed in many papers and we refer the reader to the original literature (Ref. 3 and references therein). The transport and magnetism was recently subjected to a coherent-potential approximation (CPA) treatment.⁴ The CPA is a mean-field theory which can handle delocalized states up to the short mean free path or diffusive limit, but it does not reproduce localization. Localized states and hopping transport have to be treated separately, as shown below.

Near the band edges, the eigenstates of one-particle systems are localized for energies up to the mobility edges ε_c .⁵ The localization starts when the density of states is below a critical value determined by the diffusivity and localization length.⁶ The nature of the mobility edges in disordered magnets is expected to be more complex than that of nonmagnetic alloys, but many of the usual features should remain valid. In a magnet such as GaMnAs, we now have two spin-split bands, and two mobility edges. Though conductivity, magnetoresistance, and Hall coefficient have been measured in the insulating limit,^{7,8} the dependence of the mobility edges on magnetism has not been investigated. Neither is it clear from the present understanding of the data whether all transport coefficients are truly in the hopping limit, or whether there is a substantial band-edge-delocalized contribution which looks like hopping. One usually says that transport is in the hopping limit when the measured conductivity scales with temperature as $\exp[-(T_0/T)^{1/4}]$. Here, T_0 is the renormalized Mott temperature $T_0 = 24 \times 2.7 \alpha^3 / [\pi \rho(\varepsilon_f) k_B]$, with α denoting the inverse localization length,⁵ k_B the Boltzmann constant, $\rho(\varepsilon_f)$ the density of states at the Fermi level, and 2.7 the percolation factor. However, the hopping transport with Coulomb correlation gaps often scale as $\exp[-(T_0/T)^{1/2}]$, and there are many situations in which materials exhibit $\exp[-(T_0/T)^\mu] \rightarrow 0 < \mu < 1$ laws, which are obviously not due to variable range hopping (VRH) between lo-

calized states (see Ref. 9). Emin,¹⁰ for example, has shown that small polarons will also produce such behavior for reasons not related to VRH. Indeed, one can say that almost all disordered insulators exhibit such a weaker-than-exponential law. Often it is due to granularity and Coulomb correlations.^{9,11} Furthermore, it is often not easy to distinguish between $\mu \sim 1/3, 1/4, \dots$, etc., because the data are collected over a limited range. So one has to be cautious and examine other possible scenarios. This is what we propose to do here.

We do this by presenting a complete theory of magnetotransport which is useful for other materials as well. However, as an application of the formalism developed here, we concentrate our analysis on Mn-doped GaAs. On the basis of experimental data for magnetoresistance and thermopower, we question the assumption of Mott hopping in insulating GaMnAs and we offer an alternative explanation. Concerning the Hall effect, we recall that Allen *et al.*⁸ showed that the anomalous Hall effect (AHE) in the insulating regime of GaMnAs is not only anomalous in the usual sense because it scales with magnetism but also has an anomalous sign. An explanation of this anomaly is apparently contained in the hopping Hall-effect model of Burkov and Balents.¹² Here, we propose an alternative theory which includes both hopping and band-edge conduction.

This paper is structured as follows. We first present the basic definition of the Hall effect in magnets. The processes that explain transport in the insulating limit are then discussed. All the main contributions to the Hall effect are then introduced, including hopping, intrinsic, and extrinsic, and the behavior near the mobility edge. We then discuss the magnetoresistance and the different contributions to it are introduced. Following that, all the contributions to the transport properties are summarized and discussed in relation with experiments. Finally, we present a discussion of our results. We argue that transport in the insulating regime of the magnetic semiconductors of the GaMnAs type above 10 K is mainly one of delocalized band-edge conduction.

II. HALL EFFECT

The experimentally measured Hall coefficient R_H is often written as

$$R_H = R_N + R_A,$$

$$R_A = (a\rho_{xx} + b\rho_{xx}^2)M_z/B_z^{\text{ext}}, \quad (2)$$

where a and b are constants and ρ_{xx} is the resistivity. B_z^{ext} is the external magnetic induction, defined as $\mu_0 H_z^{\text{ext}}$, where H_z^{ext} is the external magnetic field. The first term R_N is the normal Hall coefficient and scales with resistivity in the usual way and the second R_A is the anomalous term which, in general, can have two components, one linear and the other quadratic with resistivity,³ and is proportional to the magnetization M_z . The general relation for the Hall coefficient is¹³

$$R_H = \frac{\rho_{yx}}{B_z^{\text{ext}}}, \quad (3)$$

with

$$\rho_{yx} = \frac{-\text{Re}\{\sigma_{xy}\}}{\text{Re}\{\sigma_{xx}\}\text{Re}\{\sigma_{yy}\} - \text{Re}\{\sigma_{yx}\}\text{Re}\{\sigma_{xy}\}}, \quad (4)$$

where σ_{xy} and σ_{xx} denote the transverse and normal conductivities, respectively. We may suppose that as there is no voltage applied in the y direction $\sigma_{yx}\sigma_{xy} \ll \sigma_{xx}\sigma_{yy}$. Also, if we consider an isotropic system in the xy plane $\sigma_{xx} = \sigma_{yy}$ and thus we may write the Hall coefficient as^{4,14}

$$R_H = \frac{\text{Re}\{\sigma_{xy}(B_z)\}}{B_z^{\text{ext}}[\text{Re}\{\sigma_{xx}(B_z)\}]^2}. \quad (5)$$

Karplus and Luttinger¹⁵ pointed out that the B field involves the magnetic moment of the material via the internal magnetization M_z ,

$$B_z = \mu_0[H_z^{\text{ext}} + (1 - N)M_z] \equiv B_z^{\text{ext}} + \mu_0(1 - N)M_z, \quad (6)$$

where N is the demagnetizing factor. Thus, the magnetization term is implicit in the normal contribution as a shift in the magnetic field. However this form is not normally sufficient to explain the much larger magnetization contribution observed in ferromagnets.¹⁵ Karplus and Luttinger then developed the first theory of the intrinsic AHE.¹⁵ Throughout, we shall, for simplicity, suppose a thin film with the magnetic field perpendicular to the plane and thus we take $N=1$, unless otherwise mentioned. Let us now consider the problem of conduction.

III. HALL EFFECT AND CONDUCTIVITY IN THE INSULATING LIMIT

When the Fermi level is at the band edge, Anderson localization sets in, and Fermi-level transport cannot be described using only extended states. The very low-temperature transport process is by hopping, as shown by Allen *et al.*⁸ for GaMnAs. These authors have also measured the Hall effect and, as already mentioned above, observed that there is a sign anomaly for the AHE in this regime. They argue that the transport is by hopping from localized level to localized level and not by thermally assisted hole ionization into the valence band. The magnetoresistance in the insulating regime has been measured by Van Esch *et al.*⁷ These authors constructed a model to explain the very large magnetoresistance they observed, which we will discuss later. However, no one seems to have yet made a model which consistently describes resistance, magnetoresistance, and Hall effect in the insulating regime of GaMnAs.

The ordinary phonon-assisted hopping Hall effect has already been treated rigorously and in great detail.¹⁶⁻¹⁸ The extraordinary contribution reported recently by Allen *et al.*⁸ in Mn-doped GaAs can be modeled as follows. We consider p holes in the valence-band spin-orbit coupled as in the models of Baldareshi and Lipari¹⁹ and Fiete *et al.*²⁰ The holes are bound to a negatively charged, spin 5/2, Mn ion. Fiete *et al.*²⁰ also included the AFM exchange coupling of the hole spin to the d spin.

The Hamiltonian describing the dynamics of holes localized around the Mn sites and which incorporates the spin-orbit coupling to first order is given by

$$\begin{aligned}
H = & \sum_{i,s} \varepsilon_{is} c_{is}^\dagger c_{is} + \sum_{i,j,s} t_{ij} (B_z + B_{so}) c_{is}^\dagger c_{js} \\
& - \sum_{i,s,s'} \lambda_i \langle i,s | \mathbf{l}_i \cdot \boldsymbol{\sigma} | i,s' \rangle c_{is}^\dagger c_{is'} \\
& - \sum_i \mu_B (l_{i,z} + g\sigma_{i,z}) (B_z + h_{z,\text{spin}}) c_{is}^\dagger c_{is}. \quad (7)
\end{aligned}$$

In Eq. (7), the summation indices include the band and site indices, i.e., $j=j, \gamma$, where γ denotes the orbital, and we again assume $N=1$. ε_{is} are diagonal site energies. Now consider the second term, where $t_{ij}(B_z + B_{so})$ is the site-to-site transfer term. The magnetic field is incorporated in the Peierls phase. We recall that in the presence of the magnetic field we have the usual Peierls phase factor,

$$t_{mn} = t_{mn}^0 e^{-ie/2\hbar \mathbf{B}^{\text{ext}} \cdot (\mathbf{R}_n \times \mathbf{R}_m)}. \quad (8)$$

The nonlocal spin-orbit terms are not specifically included in Eq. (7) but can be reduced, in first order, to a shift of the total magnetic field by an amount B_{so} , which is included in t_{ij} and which we consider in detail below. The atomic spin-orbit coupling λ_i , in the third term in Eq. (7), mixes angular momentum \mathbf{l} and spin bands on the same atom. Assuming, for the sake of argument, that we have orbital states which are p states with $l_z = \{1, 0, -1\}$ or combinations thereof, then the coupling will flip, for example, $|1, -1/2\rangle \rightarrow |-1, 1/2\rangle$, and back again. The z component of the atomic spin-orbit term ($-\lambda l_{i,z} \sigma_z$) can be included in the sum of the Zeeman and Curie-Weiss spin splitting energies for the carriers, where μ_B is the Bohr magneton. This sum is denoted by $h_{z,\text{spin}}$ in the fourth term of Eq. (7).

The spin-orbit Hamiltonian in tight binding was examined by Pareek and Bruno²¹ for disorder induced coupling, by Movaghgar and Cochrane¹⁴ for longitudinal and Hall transports, and by Arsenaault *et al.*^{4,13} for CPA. In weak spin-orbit coupling, the effect can be incorporated into a two-site overlap phase similar to the Peierls phase given by Eq. (8).^{4,14,22} In the hopping limit, with no external field, the electric field at a site i (multibands are implicit if necessary) is given by the sum of the fields emanating from all neighbors,

$$\mathbf{E}_{\text{so},i} = \left\langle i \left| \sum_n \frac{eZ_n(\mathbf{r} - \mathbf{R}_n)(1 - p_n)}{4\pi\varepsilon\varepsilon_0|\mathbf{r} - \mathbf{R}_n|^3} \right| i \right\rangle. \quad (9)$$

In Eq. (9), ε is the dielectric constant which screens the acceptor site potential, Z_n is the effective charge, and the factor $(1 - p_n)$ is the probability that the site n is empty and therefore charged. In the i to j jump, the third closest site in the pathway will contribute an electric field which produces a net magnetic field. So the sum in Eq. (9) can be approximated to include the best neighbor to i (see Refs. 6, 17, and 23 and references therein for the definition of best neighbor). We can now include this spin-orbit phase in the magnetic phase by adding to B^{ext} in Eq. (8) the simplified effective spin-orbit field,

$$\mathbf{B}_{\text{so}}(i \rightarrow j) = \frac{\hbar}{2mc^2} \left\langle i \left| \sum_{n,n \neq i,j} \frac{Z_n(1 - p_n)}{4\pi\varepsilon\varepsilon_0|\mathbf{r} - \mathbf{R}_n|^3} \right| i \right\rangle \boldsymbol{\sigma}, \quad (10)$$

where the nearest-neighbor sum excludes the start site i and end site j . A more complete derivation for the effective spin-orbit field is presented for the case with no screening (no ε in the equation) and where all potentials contribute (no $1 - p_n$ in the equation) in Ref. 4. Note that the integral $\langle i | \frac{1}{|\mathbf{r} - \mathbf{R}_n|^3} | i \rangle$ is strictly speaking not convergent. However, in practice, the orbit radius is never allowed to be smaller than the effective atomic orbit of the valence state so that the cubic singularity does not occur.

Before determining the various contributions to the Hall conductivity, let us examine the effective masses which may appear in the expressions which follow so that the meaning is clear. There are three effective masses in the problem:

- (1) the Kane-Luttinger mass (m^*) which controls the kinetic energy in the starting Hamiltonian (effective-mass Hamiltonian);
- (2) the spin-orbit mass which is equal to the bare electron mass; and
- (3) an effective mass which is a result of the sum rule, as discussed in Refs. 24 and 25, and which can become the diffusivity in strong disorder but is the Kane-Luttinger mass in weak disorder.

A. Hopping contribution

The normal Hall term arising from the external $B = B_z$ term has been derived by the hopping interference method in the pure diffusive limit and in the VRH limit^{12,17,26} and applied to the case of phonon-assisted hopping and percolation in a one-band model. Thus, we may add the spin-orbit magnetic field to the external field and carry out the configurational average as before. So we replace B_z by

$$B_z^{\text{ext}} + \mu_0(1 - N)M_z + B_{so}, \quad (11)$$

where

$$B_{so} = \left\langle i \left| \frac{\hbar}{2mc^2} \sum_{n,n \neq i,j} \frac{Z_n(1 - p_n)}{4\pi\varepsilon\varepsilon_0|\mathbf{r} - \mathbf{R}_n|^3} \right| i \right\rangle \langle \sigma_z \rangle \equiv \lambda_{\text{hop}} \langle \sigma_z \rangle, \quad (12)$$

where $\langle \sigma_z \rangle$ is the polarization of the holes and M_z is the magnetization of the sample. The analysis can now be done as previously. The high-density hopping system, for example, has been treated analytically in Ref. 17 and the phonon-assisted percolation regime in Refs. 16, 18, and 23.

The second term in Eq. (11) depends on the geometry of the sample and is zero in thin films under normal fields.¹⁵ The sum in the third term runs over the third neighbor in the triad and is configurationally averaged to the optimum second-nearest-neighbor distance (as shown in Ref. 16), which is $|\mathbf{R}_i - \mathbf{R}_n| = \langle R_3(T) \rangle \propto (\frac{T_0}{T})^{1/4}$. Here, T_0 is the renormalized Mott temperature, defined previously.

Replacing Eq. (11) in the expression of Ref. 23 allows us to calculate the Hall mobility (sign is negative for electrons

and positive for holes) and conductivity as was done before.²³ Assuming a constant density of states, we obtain

$$\mu_{xy} = \frac{\text{Re}\{\sigma_{xy}\}}{B_z^{\text{ext}} \text{Re}\{\sigma_{xx}\}} = e^{-(3/8)(T_0/T)^{1/4}} \left[1 + \frac{\langle \sigma_z \rangle}{B_z^{\text{ext}} \lambda_{\text{hop}}} \right], \quad (13)$$

$$\text{Re}\{\sigma_{xx}\} = \sigma_0 e^{-(T_0/T)^{1/4}} = \frac{1}{\text{Re}\{\rho_{xx}\}}, \quad (14)$$

and

$$\frac{\rho_{xy}}{B_z^{\text{ext}}} = \frac{\mu_{xy}}{\text{Re}\{\sigma_{xx}\}} = e^{(5/8)(T_0/T)^{1/4}} \left[1 + \frac{\langle \sigma_z \rangle}{B_z^{\text{ext}} \lambda_{\text{hop}}} \right]. \quad (15)$$

Here ρ_{xy} is the Hall resistivity, σ_0 is the conductivity prefactor, and we have used $N=1$. Note that for the remainder of the paper we will no longer distinguish the real part of the conductivity from total conductivity since they are the same at zero frequency.

The factor 3/8 in Eq. (13) is a consequence of averaging over the third site in the triad $\langle 2\alpha R_3 \rangle \sim \frac{13}{8} (\frac{T_0}{T})^{1/4}$. When the density of states is quadratic in energy, rather than constant, we replace $(\frac{T_0}{T})^{1/4}$ by $(\frac{T_0}{T})^{1/2}$. This is also true for the Coulomb gap hopping problem.^{16,23,27,28}

Allen *et al.*⁸ observed an experimental $\exp[-(\frac{T_0}{T})^{1/2}]$ dependence of conductivity. This is characteristic of a Coulomb gap or hopping in a quadratic density of states. They also found a near $\rho_{xx}^{1/2}$ behavior of the Hall resistivity. This is also reproduced by the present theory, as shown in Eq. (15). Finally, they found normal and anomalous processes which have opposite signs. The sign anomaly can be explained by the sign of $\langle \sigma_z \rangle$ which can be opposite to the overall magnetization since it refers only to carriers at the Fermi level.

The maximum strength of the AHE predicted by Eqs. (13)–(15) is determined by the magnitude λ_{hop} . An estimate, which includes the factor 1000 reduction due to the cube of the average distance to the third neighbor, gives $\sim 3 \times 10^3$ G. This spin-orbit term is somewhat too small at $T=25$ K when the result is compared to the experimental data in Ref. 8. The data suggest that the AHE is more important than the normal Hall effect, even at an external field of 10 T. The measured order of magnitude is close to the theory estimate only in the truly very low-temperature hopping regime where $\text{AHE} \sim \text{NHE}$, i.e., when $T \sim 10$ K or less. The ratio of AHE to NHE is always roughly 5 or 6 when we look at the data in the *metallic regime*. It seems therefore that though the simple hopping model may be right for the truly low-temperature hopping regime $T < 10$ K at higher temperatures the spin-orbit electrical fields which are needed to produce the sideways force are larger than those predicted by hopping theory.

This is possibly because, in this regime, the Hall charge transport is, in fact, already activated to above the mobility edge and is not really hoppinglike. We shall examine this possibility in Sec. III B. This view is also supported by magnetoresistance⁷ and thermopower data²⁹ (although Osinny *et al.*²⁹ said otherwise). Thus, the picture of the third site in the triad providing the spin-orbit field is attractive, but the effect produced by this mechanism is probably not large

enough to explain the quantitative values. States excited above the mobility edge undergo skew scattering and intrinsic Hall scattering and feel the full effect of the lattice-induced spin-orbit enhancement, as discussed by Chazalviel,³⁰ Berger,³¹ Fivaz,³² and Engel *et al.*³³ The previous work of de Andrada e Silva *et al.*,³⁴ for example, gives a maximum enhancement factor of the bare coupling of the form

$$\gamma_{\text{enh}} = \left(\frac{m}{m^*} \right) \frac{mc^2}{E_g} \frac{\Delta(2E_g + \Delta)}{(E_g + 3\Delta)(3E_g + \Delta)}, \quad (16)$$

where Δ is the Kane spin-orbit coupling energy, E_g is the energy gap of the semiconductor, and m^* is the effective mass. One can see that in GaAs, the enhancement of the spin-orbit coupling is considerable. Thus, the pathways above the mobility edge, even if not necessarily always dominant for the ordinary transport, can be expected to be dominant for the AHE. It is difficult to see where such an enhancement would come from in hopping conduction. Including the many orbital bands will not strongly affect the results in the hopping limit. The marginal but important contributions from the hopping pathways forces us to now look for other possible explanations or other contributions to the hopping channel especially at higher temperatures.

B. Intrinsic Luttinger contributions from the region above the mobility edge

Consider the contributions to transport coming from excited states below (for holes) the mobility edge. For delocalized states, there are skew scattering Hall contributions due to impurity potentials. This has been treated in Refs. 33 and 35. There is also an intrinsic Luttinger contribution which can be written, in the language of Chazalviel,³⁰ and as re-derived to first order in spin-orbit coupling by Arsenault and Movaghar.²⁵ We write the transverse conductivity as

$$\sigma_{xy}^j = \frac{e^2}{\Omega} \frac{1}{e} \sum_{\alpha} \left(-\frac{\partial f(\epsilon_{\alpha})}{\partial \epsilon_{\alpha}} \right) \mu_B \sigma_z^{\alpha} \Delta g_{\alpha}^{zz}, \quad (17)$$

where

$$\sigma_z^{\alpha} = \frac{n_{\alpha\uparrow} - n_{\alpha\downarrow}}{n_{\alpha\uparrow} + n_{\alpha\downarrow}}, \quad (18)$$

$e = -|e|$ for electrons, and $e = |e|$ for holes. Finally, the zz component of the effective g -shift tensor is

$$\Delta g_{\alpha}^{zz} = \sum_{\beta} \left\langle \alpha \left| \frac{\hbar}{4m^2c^2} [\nabla V(\mathbf{r}) \times \mathbf{p}]_z \right| \beta \right\rangle \frac{1}{\epsilon_{\beta} - \epsilon_{\alpha}} \langle \beta | L_z | \alpha \rangle. \quad (19)$$

In the preceding equations, $\mu_B = \frac{e\hbar}{m}$, $f(\epsilon_{\alpha})$ is the Fermi function, $n_{\alpha s}$ is the number of electrons in the state α with spin s , and Δg_{α}^{zz} is the zz component of the g -shift tensor. This component can be related to the result first derived by Karplus and Luttinger¹⁵ by writing approximately $\Delta g \sim \lambda / \Delta_{\text{gap}}$, where Δ_{gap} is a (Bloch) subband gap and λ is the spin-orbit coupling strength. The g -shift tensor is used here as in the language of Chazalviel³⁰ and includes the Bloch band en-

hancement. Note that the intrinsic contribution given by Eq. (17) apparently does not scale with the conductivity. However, this is only true in a highly ordered medium in which the scattering linewidth is smaller than the intersubband energies. In this limit, it only appears as a lifetime term in a basis of well-defined Bloch states. In a strongly disordered system, there are no well-defined Kohn-Luttinger bands as such, and the g -shift tensor will contain the relaxation-time term in the denominator as well. When the linewidth is larger than the dominant excitation energies in the sum, then the relaxation time will enter the g -shift tensor and the Luttinger term will also scale with resistance.

C. Magnetic field-dependent resistance and Hall-effect contributions from the region above the mobility edge

We can now give a more complete formula which includes the hopping channel and the mobility edge contributions and which allows us to predict the magnetoresistance as well. From the standard mobility edge transport theory^{5,36} and including the normal Hall effect, the skew scattering contribution,^{33,35} and the intrinsic Karplus-Luttinger term, we have

$$\sigma_{xy} = e^2 \int_{-\infty}^{\varepsilon_c} d\varepsilon \left(-\frac{\partial f_h(\varepsilon)}{\partial \varepsilon} \right) \rho(\varepsilon) D_{\text{diff}}(\varepsilon) \left(\frac{eB_z}{m(\varepsilon)} \tau(\varepsilon) + \langle \sigma^z \rangle \frac{\tau(\varepsilon)}{\tau_s(\varepsilon)} \right) + \sigma_{xy, \text{hopping}}(T) + \sigma_{xy}^i, \quad (20)$$

where the so-called intrinsic AHE is

$$\sigma_{xy}^i = e^2 \int_{-\infty}^{\varepsilon_c} d\varepsilon \left(-\frac{\partial f_h(\varepsilon)}{\partial \varepsilon} \right) \rho(\varepsilon) \frac{\hbar}{m^*} \Delta g(\varepsilon) \sigma^z(\varepsilon), \quad (21)$$

and

$$\sigma_{xx} = e^2 \int_{-\infty}^{\varepsilon_c} d\varepsilon \left(-\frac{\partial f_h(\varepsilon)}{\partial \varepsilon} \right) \rho(\varepsilon) D_{\text{diff}}(\varepsilon) + \sigma_{\text{hopping}}(T). \quad (22)$$

The second part of the first term in Eq. (20) is the skew scattering contribution with $\frac{1}{\tau_s}$ denoting the skew scattering rate. Here, $f_h(\varepsilon)$ is the hole distribution. In Eqs. (20) and (22), $D_{\text{diff}}(\varepsilon)$ is the band diffusivity at energy ε and, by localization theory,³⁶ is of the form

$$D_{\text{diff}}(\varepsilon) = D_0 \left| \frac{\varepsilon_c - \varepsilon}{\varepsilon_c} \right| \quad (23)$$

near the mobility edge. Note that the Luttinger term contains the quantum diffusivity \hbar/m^* and not the actual diffusivity D_{diff} . In Eq. (23) $D_0 \sim 1 \text{ cm}^2/\text{s}$, $\rho(\varepsilon)$ is the density of states for which one normally adopts an exponential form

$$\rho(\varepsilon) = \rho_0 e^{-\varepsilon/\varepsilon_0}, \quad (24)$$

with $0 < -\varepsilon < -\varepsilon_c$, where ρ_0 is the value at the Fermi level and ε_0 measures the exponential steepness. The spin-orbit coupling is contained in the g -shift factor. If the intersubband gaps Δ_{gap} is definable, the g -shift tensor is $\Delta g_\alpha \sim \frac{\lambda}{\Delta_{\text{gap}}}$ as in

the paper of Karplus and Luttinger.¹⁵ In highly disordered alloys, there are no well-defined subband gaps, as assumed by Karplus and Luttinger¹⁵ and Jungwirth *et al.*³ Then, one has to evaluate the g -shift tensor more rigorously. The g value of the Mn spin+hole complex (~ 1.98) has been measured by ferromagnetic resonance and is in the range of -0.05 – -0.2 , *holelike* ($\Delta g < 0$), and increases with Mn-hole doping (see the review by Liu and Furdyna³⁷). Note that the sign anomaly can again, as in hopping, be due to the fact that the holes at the Fermi level are polarized opposite to the bulk magnetism.

The hole Fermi function $f_h(\varepsilon)$ is, to a good approximation, a Boltzmann function when $|\varepsilon_f - \varepsilon_c| \gg k_B T$. The mobility edge for holes is denoted by ε_c , and the energy dependence of $\rho(\varepsilon) D_{\text{diff}}(\varepsilon)$ dominates the longitudinal conductivity. The anomalous skew scattering Hall contribution will be dominated by the spin-orbit band term via $\frac{1}{\tau_s(\varepsilon)}$ in Eq. (20).^{33,35} The skew scattering contribution is not negligible because of the very large spin-orbit coupling enhancement (10^4), implicit in Eq. (16), when applied to GaMnAs. It is useful to recall that Engel *et al.*³³ estimated the skew factor to be $\frac{\tau}{\tau_s} \sim \frac{1}{900}$ for Si-doped GaAs. Thus, when comparing $\frac{\tau}{\tau_s} \langle \sigma^z \rangle$ with $\omega_c \tau$ and noting that $\tau \sim \frac{Dm^*}{|\varepsilon_c - \varepsilon|}$, $\varepsilon < \varepsilon_c$ (for holes), we see that the skew scattering band contribution to the AHE is comparable to the ordinary Hall effect even near band edges. The Luttinger term [Eq. (21)] is $\sim \{10^{-4} - 10^{-3}\} \times \Delta g \langle \sigma^z \rangle$ and will dominate when $\Delta g > 10^{-3}$. We return to this later. The mobility edge channel gives rise to very concrete formulas for the behavior of the magnetoresistance and Hall effect. So now we should examine the hopping magnetoresistance and then we may be able to decide which mechanism dominates in each range of B field and temperature.

IV. MAGNETORESISTANCE

A. Hopping magnetoresistance

The relative importance of the hopping and band-edge weak localization contributions could, in principle, be tested by measuring the corresponding magnetoresistance. We shall now show that these are very different, both in structure and in order of magnitude. It is surprising that magnetoresistance was not measured by Allen *et al.*⁸ The magnetoresistance in the apparently *hoppinglike regime* was measured in a similarly low doped (2%) system by Van Esch *et al.*,⁷ who found a strong negative magnetoresistance, which they interpreted as being due to the orbital expansion of the localized states in a B field. This, the authors suggested, is caused by the competition between the spin Zeeman energy and the antiferromagnetic hole-Mn coupling. The external field is, however, far too small to interfere with the strong antiferromagnetic J_{pd} coupling of $\sim 0.5 \text{ eV}$.³ The magnetoresistance data of Van Esch *et al.*⁷ actually look very similar to those obtained for metallic films by Matsukura *et al.*³⁸

Let us now examine the complete hopping magnetoresistance and then evaluate whether it can explain the available data⁷ in the insulating regime. The theory will also allow us to make predictions concerning what should be the behavior of the hopping magnetoresistance. Specifically, hopping spin

and orbital MR do exist^{39–41} and have been discussed by Movaghar and Schweitzer⁴² as applied to *a*-Si. Hopping magnetoresistance usually has a positive contribution from orbital shrinking of wave functions at very high field, $B > 1$ T (Mikoshiba effect). This is a one-body effect. Localized states also have a finite Hubbard U , so there is also a two-body contribution to the spin-hopping magnetoresistance. An electron from below the Fermi level can, in general, hop to an empty state, or one which is already occupied by another electron below. The doubly occupied level is shifted up by the Hubbard U and, if U is not too large, this takes some of the doubly occupied states to the Fermi level. The jump to the occupied state takes place in the triplet or singlet configuration. The triplet hop being forbidden, the three triplet states would have to first convert into singlets. This process depends sensitively on (small) transverse (B_x, B_y) magnetic fields within the system.⁴² The triplet-to-singlet transformation can also occur during the jump via the spin-orbit coupling $\lambda \mathbf{l} \cdot \boldsymbol{\sigma}$. We can write the totality of effects as⁴²

$$\sigma_{xx}^{\text{hop}}(B_z, T) = \sigma_0 e^{-(T_0/T)^{1/4}(1+\xi(eB_z/\hbar)^2)} \times \left[N_e + N_S + N_T \left(\frac{\nu_{\text{so}}}{\nu_0} + \frac{\left(\frac{\mu_B \langle B_{\text{int}}(0) \rangle}{\hbar} \right)^2 \tau^2}{(\omega_c \tau)^2 + 1} \right) \right]. \quad (25)$$

In Eq. (25), ξ is a quantity that will be given below. N_e, N_S , and N_T are probabilities that the hop is to a previously empty, singlet, or triplet state. N_S and N_T depend on the magnetic field as $N_S = \frac{1-\langle \sigma_z \rangle}{4}$ and $N_T = \frac{3+\langle \sigma_z \rangle}{4}$. The direct spin-orbit-assisted triplet-to-singlet hop is proportional to $\frac{\nu_{\text{so}}}{\nu_0}$, where ν_0 is the hopping (singlet) prefactor. The internal random fluctuating magnetic fields $B_{\text{int}}(t)$ which allow spin admixtures to occur are mainly due to fluctuations in g^{xz} and g^{yz} and the anisotropic part of the dipole-dipole interaction. For weak coupling, we have

$$\langle B_{\text{int}}^2 \rangle = \langle B_{\text{dip}}^2 \rangle + \frac{1}{2} [|\Delta g_{xz} B_z^{\text{ext}}|^2 + |\Delta g_{yz} B_z^{\text{ext}}|^2]. \quad (26)$$

The dipolar internal fields $\langle B_{\text{dip}}^2 \rangle^{1/2}$ are of the order of the ESR linewidths smaller than 10 mT; the g admixture is $\sim 10^{-3}$. Saturation occurs when the frequencies corresponding to Eq. (26) are comparable with typical hopping frequencies. A similar model has been used recently to explain the magnetoelectroluminescence in polymers.⁴³

The Mikoshiba effect, mentioned earlier, is spin independent and changes the phonon-assisted hopping rate as

$$W_{ij} = W_{ij}(0) e^{-2\alpha R_{ij} - R_{ij}^3 / 6\alpha (eB/\hbar)^2 - \varepsilon_j - \varepsilon_i / k_B T}, \quad (27)$$

averaged over the angle between the B field and \mathbf{R} and with $\varepsilon_j > \varepsilon_i$. At optimum hopping length, R_{ij}^3 goes to $\langle (\alpha R)^3 \rangle = (\frac{3}{8})^{3/4} (\frac{T_0}{T})^{3/4}$ and $\frac{|\varepsilon_j - \varepsilon_i|}{k_B T}$ goes to $\frac{1}{4} (\frac{T_0}{T})^{1/4}$. Inserting these terms in Eq. (27) gives us $\xi = (\frac{T_0}{T})^{1/2} (\frac{3}{8})^{3/4} \frac{1}{6\alpha^4}$ in Eq. (25). To lowest order, we may neglect the change in T_0 caused by B_z . There is also an increase in the energy of the localized state. This

shift lowers the ionization energy to the mobility edge which, in first-order perturbation theory, is

$$\varepsilon_i(B_z) = \varepsilon_i + \frac{(eB_z)^2}{2\alpha_i^2 m}, \quad (28)$$

where α_i is the inverse localization length of the i th level. Finally, there is also a small quantum interference magnetoresistance contribution due to the fact that the hopping carrier could also virtually visit the second best neighbor site on the way to its best neighbor. This contribution has been analyzed in detail by Schirmacher.⁴⁴ The spatial part of the transfer rate is, to first order, proportional to

$$|T_{ij}(B)|^2 = \left| t_{ij}^0 e^{-ie/\hbar \mathbf{B} \cdot (\mathbf{R}_i \times \mathbf{R}_j)} + \sum_l \frac{t_{il}^0 t_{lj}^0}{\varepsilon_i - \varepsilon_l} e^{-ie/\hbar \mathbf{B} \cdot (\mathbf{R}_i \times \mathbf{R}_l + \mathbf{R}_l \times \mathbf{R}_j)} \right|^2. \quad (29)$$

This now includes the direct transfer and the interference term from a third site. This means that the Miller-Abrahams⁴⁵ hopping spatial factor, $|t_{ij}^0|^2$, should be replaced by Eq. (29) which is a small but magnetic-field-dependent factor. One need take only one term in the l sum, the second-nearest-neighbor term. The optimized second neighbor $2\alpha R_{il}$ and $\frac{\varepsilon_l - \varepsilon_i}{k_B T}$ will also scale as $\gamma (\frac{T_0}{T})^{1/4}$ but with a constant factor γ , which instead of the first neighbor values, 3/4 and 1/4, respectively, will be somewhat larger, $\sim 39/32$ for the spatial length and 13/32 for the energy. We have already used this for the hopping Hall effect [Eq. (13)]. The sign depends on the geometry of the triad and can be positive or negative.

We conclude that in Eq. (25), we have to replace σ_0 by $\sigma_0 \langle | \frac{T_{ij}(B_z)}{t_{ij}} |^2 \rangle$, where $\langle \rangle$ is the spatial/energy average over the best second neighbor (see Refs. 17 and 23 and references therein for the definition of best second neighbor). Keeping the best third neighbor l and assuming $R_l=0$, we obtain a quantum interference contribution,

$$\Delta \sigma_0(B) = \sigma_0 \left\langle \frac{2t_{il}t_{lj}}{t_{ij}(\varepsilon_i - \varepsilon_l)} \left[\cos \left(\frac{e}{\hbar} \mathbf{B} \cdot (\mathbf{R}_l \times \mathbf{R}_j) \right) - 1 \right] \right\rangle. \quad (30)$$

The average in Eq. (30) is over the percolation network and gives a negative magnetoresistance, according to Schirmacher.⁴⁴ In general, one has to note that all hopping contributions below 1 T are at best a few per cent and nowhere near the large values observed by Van Esch *et al.*⁷ This completes the hopping analysis. Now consider the contribution to the magnetoresistance coming from excitations to the band edge.

B. Orbital magnetoresistance from the region near the mobility edge

Following Khmelitskii *et al.*³⁶ and Movaghar and Roth,⁴⁶ we have for holes and small B fields the following shift for the mobility edge:

$$|\varepsilon_f(B_z) - \varepsilon_c(B_z)| \sim \varepsilon_f(0) - \varepsilon_c(0) \left[1 + \kappa \left(\frac{\hbar \omega_c}{\varepsilon_c(0)} \right)^{1/2} \right], \quad (31)$$

where κ is a constant of order 1 and ω_c is the cyclotron frequency of the holes. In Eq. (31), we have neglected the magnetic-field shift in the Fermi energy compared to that of the mobility edge. This result is valid for fields at which, $\hbar \omega_c < \hbar / \tau$, cyclotron energy is less than the scattering broadening and gives, from Eq. (31), an $\exp\left[\frac{\kappa[\hbar \omega_c \varepsilon_c(0)]^{1/2}}{k_B T}\right]$ increase in the conductivity over a limited field range.

The above describes the well-known orbital delocalization effect. The fully self-consistent shift in the effective activation energy, i.e., the exact value of the one-body $\varepsilon_f(B_z) - \varepsilon_c(B_z)$ shift, which includes the ferromagnetic effects, and high B fields is not known and this would need a detailed study of localization in a disordered ferromagnet. However, it can be inferred that in the present case, field-induced magnetic alignment is going to lower disorder and thus increase the range of delocalized states by shifting $\varepsilon_c(B_z, M_z)$ up with M_z and $\varepsilon_f(B_z)$ down with B_z . At low temperatures, the magnetism can be saturated at relatively low fields and will not change the electronic structure very much below T_c .

V. SUMMARY OF CONTRIBUTIONS INCLUDING SPIN-DISORDER SCATTERING

We now have a theory which allows us to calculate, in principle, the resistance, magnetoresistance, and Hall coefficient when the Fermi level is in the region of localized states. We have also included the band region at, or just above, the mobility edge including the spin scattering effects which can be described by the CPA. This is discussed in Appendix C. The Hall conductivity has already been given by Eq. (20).

Consider first the complete longitudinal conductivity in the presence of a magnetic field B_z and including orbital and spin contributions from localized and delocalized states, this is given by

$$\begin{aligned} \sigma_{xx} = & \frac{e^2}{\pi} \int_{\varepsilon_b}^{\varepsilon_c(B_z)} d\varepsilon \left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \rho_{\text{edge}}(\varepsilon) D_0 \left| \frac{\varepsilon - \varepsilon_c(B_z)}{\varepsilon_c(B_z)} \right| \\ & + \int_{-\infty}^{\varepsilon_b} d\varepsilon \left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \rho_{\text{band}}(\varepsilon) D_{\text{CPA}}(\varepsilon, B_z) + \sigma_{xx}^{\text{hop}}(B_z), \end{aligned} \quad (32)$$

where ε_b is the energy above which the CPA stops being applicable. This energy is close to, but a little bit below, the mobility edge (we are considering hole). This expression includes the band edge (first term), the high mobility band term (second term), and the third term, which is the hopping contribution. In the first, or band-edge term, the magnetic-field dependence is basically due to the B -field-induced delocalization and not to spin. In the second, or band term, which takes over at higher temperatures, the magnetoresistance is mainly due to the spin-disorder scattering and is negative because the magnetic field reduces the disorder and thus reduces the scattering rate. Figures 1 and 2 are examples of this

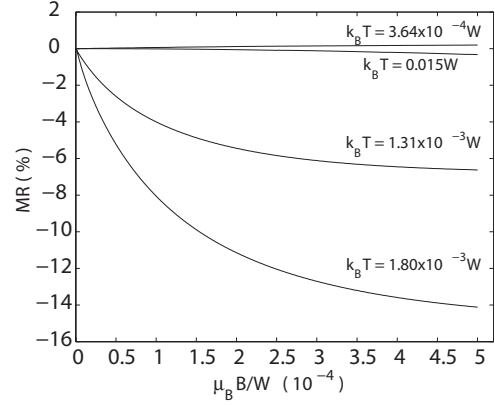


FIG. 1. Relative magnetoresistivity in the CPA for carrier concentration $p=0.1x$ for impurity concentration $x=0.053$ and $J_{pd}=0.35 W$, with T as a parameter. W correspond to half the bandwidth. Figure was taken from Ref. 4. This is the expected behavior just below (hole) the mobility edge.

behavior. Unfortunately, we do not have the analytical form for this term, but we have the numerical CPA results shown in Figs. 1 and 2.

Also, from Eq. (15) of Ref. 7, we can create an approximation to the CPA from perturbation theory. By defining the pd scattering time as

$$\begin{aligned} \frac{1}{\tau_{pd}} = & \frac{J_{pd}^2 \rho_{\text{band}}(\varepsilon)}{\hbar} \left[S(S+1) - S^2 \left(\frac{M_z}{M_s} \right)^2 \right. \\ & \left. - S \left(\frac{M_z}{M_s} \right) \tanh \left(\frac{3T_c M_z}{2TS(S+1)M_s} \right) \right], \end{aligned} \quad (33)$$

the CPA diffusivity can be written as

$$D_{\text{CPA}} \sim v(\varepsilon)^2 \frac{\tau_{pd} \tau_e}{\tau_{pd} + \tau_e}, \quad (34)$$

where τ_e is the nonmagnetic relaxation time and $v(\varepsilon) = \sqrt{\frac{2\varepsilon}{m^*}}$ is the band velocity at energy ε . Equations (33) and (34) could

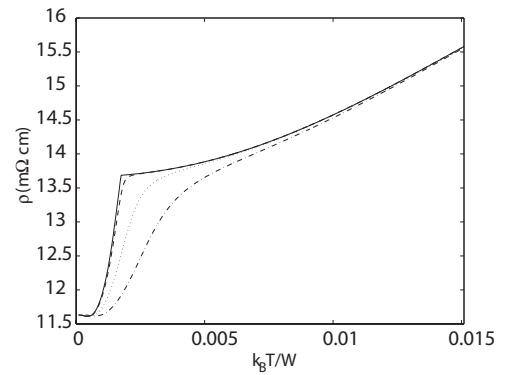


FIG. 2. Resistivity as a function of temperature for various magnetic fields. Others parameters are $x=0.053$, $E_{\text{NM}}=E_{\text{M}}=0$, $J_{pd}=0.35 W$, and $p=0.1x$. W correspond to half the bandwidth. The solid line is the zero-field result, the dashed line is for $\mu_B B=1 \times 10^{-5} W$, the dotted line is for $\mu_B B=1 \times 10^{-4} W$, and the dashed-dotted line is for $\mu_B B=3 \times 10^{-4} W$. Figure was taken from Ref. 13.

be interpreted as a first semianalytical approximation to the CPA result (see Fig. 1 for a numerical evaluation). The spin-hopping term is given by Eq. (25).

The density of states has been divided into three sections corresponding to the region near the Fermi level (hopping), the region at the band edge [Eq. (24)], and the delocalized region just below ε_c in the CPA regime. We have established that when the Fermi level is in the region of localized states, both band edge and hopping will, in general, contribute to transport. The question is to what extent is it one or the other? This obviously depends on temperature. However, it has been argued that if the observed conductivity scales as Mott's law, then the explanation cannot be anything else but VRH for all the transport coefficients. There are some facts that speak against this simple interpretation. The thermopower data are not hoppinglike.²⁹ Our estimates show that the AHE (Ref. 8) can, at very best, only be due to hopping at very low temperatures. However, the Hall data, thermopower, and magnetoresistance are all on different samples, so one has to be very careful and rigorous and examine all potential contributions.

Indeed, with the usual assumption of exponential density of states [Eq. (24)], the first term in Eq. (32) will give an activated band-edge transport behavior, so, in this limit, we have the complete result,

$$\begin{aligned} \sigma_{xx} = & e^2 D_0 \rho_0(\varepsilon_f(B_z)) \frac{k_B T}{\varepsilon_c(B_z)} e^{-\varepsilon_f(B_z) - \varepsilon_c(B_z)/k_B T + \varepsilon_c(B_z)/\varepsilon_0} \\ & + \sigma_{xx}^{\text{hop}}(T, B_z) \equiv \sigma_{xx}^{\text{BE}}(T, B_z) + \sigma_{xx}^{\text{hop}}(T, B_z). \end{aligned} \quad (35)$$

Using Eq. (21) and replacing the quantum diffusivity \hbar/m^* with the normal diffusivity, we have

$$\begin{aligned} \sigma_{xy} = & \sigma_{xx}^{\text{BE}}(T, B_z) \left[\frac{e B_z \tau(\varepsilon_c(B_z))}{m^*} + \Delta g(\varepsilon_c(B_z)) \langle \sigma_z \rangle_{\varepsilon_c} \right] \\ & + \sigma_{xy}^{\text{hop}}(T, B_z). \end{aligned} \quad (36)$$

If we had kept the original form of Eq. (21), the dependence of the Hall coefficient (R_H) on the resistance would be resistance squared as observed in most ferromagnets.

To complete the analysis we also need Eq. (31). The conclusion one can draw by looking at Eqs. (31), (35), and (36) is that as temperature increases, an activated law should be observed with a large $\exp(\kappa \varepsilon_c \sqrt{\hbar \omega_c} / \varepsilon_c / k_B T)$ increase in conductance with B (negative magnetoresistance).

A word of caution: the shift in mobility edge with B field as applied to electrons or holes assumes that the excited-state wave function has time to build an Anderson level with all its features.³⁶ This is not necessarily true for highly excited states. As an example, take a -Si:H where no such negative MR is observed despite activation above the mobility edge. In the present case, the mobility edge is very close to the Fermi level so that it is reasonable to assume that this mechanism is also operating.

Van Esch *et al.*⁷ fitted their data to a $(\frac{\hbar \omega_c}{k_B T})^{1/3}$ law. However, most often a Mott law is observed in conduction. Osinij *et al.*²⁹ analyzed the thermopower in low-doped insulating samples and observed an activated law (see Appendix B for thermopower). The resistance they measured also looks

activated, although the authors do not explicitly state it. So their data would agree with the band-edge formulas. From this, one would have to conclude that both band-edge transport and hopping are observed, depending on the sample and the transport coefficient. However, when we examine the order of magnitude of the hopping magnetoresistance we have, at most, a few percent, even at very high fields. The data of Van Esch *et al.*⁷ on magnetoresistance give us a strong clue as to what might be happening. They have a few hundred percent MR but with Mott-type conductivity. Finally, we have one last question to examine. We need to investigate what other mechanism can give us a Mott-type law for conduction but a giant magnetic bandlike magnetoresistance and AHE. Let us examine this possibility before we come to a final decision on the mechanism.

A. Mott law from correlation holes or granularity

The influence of correlations was treated by Anglada *et al.*⁹ and more recently by Dunford *et al.*¹¹ in the context of transport in polymers, granular systems, and nanoparticle composites. The point is that in highly correlated systems, the density of excited states at the band edge need not, and in general will not, be exponential. Coulomb energies and spin fluctuations can modify the excited-state spectrum. Though the exponential structure for $\rho(\varepsilon)$ is reasonably well established in systems which are weakly correlated, such as non-magnetic amorphous semiconductors,⁴⁷ one knows that in electron and spin glasses, this is not the case and the final-state interactions are important. When an empty state just above the Fermi level is filled by an electron excited from the valence band, the delocalized hole created in the valence band is not immediately neutralized. The hole which is created recombines elsewhere in the material, with an electron from below the Fermi level, and in this way, charge has been transported. The process of the electron separating itself from its hole is complex in a highly correlated spin system such as GaMnAs. Here, it is the carriers which cause the spin alignment in the first place. We can expect the density of states for excitations in such a system to be more sharply cut off at small energies. We shall see that the proposed structure will reproduce a Coulomb glass-type behavior with temperature.⁴⁸ A similar situation is encountered in granular systems^{9,46} where Mott-type laws are observed as a result of intergrain thermally or fluctuation-assisted tunneling.⁴⁹ Granularity is also expected in these materials. Indeed we can simulate this behavior with a density of states per volume of the form

$$\rho_{\text{edge}}(\varepsilon) = \rho_{\text{band}} e^{-(\varepsilon_g / \varepsilon_f - \varepsilon / \varepsilon_l)^\nu}, \quad (37)$$

characteristic of correlated or granular systems, with $0 < \nu < 1$ and where ε_g and ε_l are constants which measure steepness and the density of states at the Fermi level. Finally, ρ_{band} is a constant.

Knowing the density of states at the band edge, we can now proceed to evaluate the Hall effect, longitudinal conductivity, and magnetoresistance at the band edge. To see how we can get a Mott-type law from extended states, we use the band-edge term of Eqs. (32) and (37) to get

$$\sigma_{xx} = \frac{e^2}{k_B T} \int_{-\infty}^{\varepsilon_c(B_z)} d\varepsilon \rho_{\text{band}} D_0 \left| \frac{\varepsilon_c(B_z) - \varepsilon}{\varepsilon_c(0)} \right| \times e^{-|\varepsilon_f(B_z) - \varepsilon|/k_B T - (\varepsilon_g/|\varepsilon_f(B_z) - \varepsilon| + \varepsilon)^\nu}. \quad (38)$$

A steepest-descent evaluation of Eq. (38) at low T gives for $\nu=1/2$ an $\exp[-(T_0/T)^{1/3}]$ law (see Appendix A). Here, the Mott-type laws are a result of the pseudogap in the density of states or a depletion of low-energy excitations created by long-range correlations.

B. Quantitative comparison: VRH versus delocalized band-edge conduction

To assess the relative importance of the various channels, let us compare some values starting with σ_{xx} . The band-edge conductivity with a correlation hole density of states evaluated using steepest descent (Appendix A), and the usual hopping conductivity, would give us (with $\nu=1/3$)

$$\sigma_{xx}(\text{band}) = \frac{e^2}{\varepsilon_c(B_z) k_B T} D_0 \rho_{\text{band}} \times \{\varepsilon_{\text{max}} - [\varepsilon_f(B_z) - \varepsilon_c(B_z)]\}^2 e^{-3/2(\varepsilon_g/k_B T)^{1/4}}, \quad (39)$$

where

$$\varepsilon_{\text{max}} = (\nu k_B T \varepsilon_g^\nu)^{1/(1+\nu)}. \quad (40)$$

With Eq. (31), for holes, we obtain

$$\sigma_{xx}(\text{band}) = \frac{e^2}{\varepsilon_c(0) k_B T} D_0 \rho_{\text{band}} \times \left\{ \varepsilon_{\text{max}} - \left[\varepsilon_f(0) - \varepsilon_c(0) \left(1 + \kappa \left(\frac{\hbar \omega_c}{\varepsilon_c(0)} \right)^{1/2} \right) \right] \right\}^2 \times e^{-3/2(\varepsilon_g/k_B T)^{1/4}}, \quad (41)$$

and

$$\frac{\Delta \sigma_{xx}}{\sigma_{xx}} = \frac{2 \varepsilon_{\text{max}} \varepsilon_c(0) \kappa \left(\frac{\hbar \omega_c}{\varepsilon_c(0)} \right)^{1/2}}{(\varepsilon_{\text{max}} - [\varepsilon_f - \varepsilon_c(0)])^2}. \quad (42)$$

For the Hall conductivity, we obtain, to a good approximation ($\nu=1/3$), for holes

$$\sigma_{xy}(\text{band}) = \sigma_{xx}(\text{band}) \left[\frac{e B_z \tau(\varepsilon_{\text{max}})}{m^*} + \langle \Delta g(\varepsilon_{\text{max}}) \rangle \langle \sigma_z \rangle_{\varepsilon_{\text{max}}} \right]. \quad (43)$$

Whereas for VRH hopping (without the magnetoresistance terms) we have

$$\sigma_{xx}(\text{hopping}) = \frac{e^2}{4 \alpha^2} \rho(\varepsilon_f) \left[\frac{3}{4} \left(\frac{T_0}{T} \right)^{1/4} \right]^2 \nu_0 e^{-(T_0/T)^{1/4}}. \quad (44)$$

It is understood that the $T^{1/4}$ factors are chosen to be the same so that this determines the density of states parameters

ε_g . Thus, $(3/2)^4 \varepsilon_g/k_B = T_0$. The quantities are $\rho_{\text{band}} \sim 10^{20} - 10^{21}/\text{cm}^3 \text{ eV}$, $D_0 \sim 10^{-4} \text{ m}^2/\text{s}$, and $\rho(\varepsilon_f) \sim 10^{17} - 10^{19}/\text{cm}^3 \text{ eV}$. T_0 or ε_g is fixed by experiment. From the data of Allen *et al.*,⁸ we extract roughly $T_0 = 2.4 \times 10^5$, $T_0 = 24 \times 2.7 \alpha^3 / k_B \rho(\varepsilon_f)$. The localization radius corresponds to a binding energy of 110 meV which is $\sim 3 \text{ nm}$ for a light hole. The corresponding $\rho(\varepsilon_f) \sim 3 \times 10^{19}/\text{cm}^3 \text{ eV}$ extracted is a very large number and normally above the value required for delocalization.

One can see that the band pseudohopping expression can easily account for the dc conduction. We see that by selecting ν , one can create an $e^{-(T_0/T)^{1/2}}$ or $e^{-(T_0/T)^{1/4}}$ behavior without variable range hopping. Thus, if some experiments give one or the other law, it only means that the effective density of states is different. This has nothing to do with VRH. Furthermore, from Eqs. (31) and (42), it gives the correct order of magnitude, the right sign, and nearly the exact form of the magnetoresistance, whereas the hopping magnetoresistance is *nowhere near the right order of magnitude and structure*. The magnetism and large magnetoresistance observed by Van Esch *et al.*⁷ at very low temperature suggest that inhomogeneity and clustering play an important role. Hopping magnetoresistance would, however, seem to be the correct explanation for the magnetoelectroluminescence data in the work of Mermer *et al.*⁵⁰ where effects are a few percent for B fields of milliteslas, as in the magnetoresistance and photoconduction of a :Si.^{39,42,51}

Finally, the same optimization applied to the Hall effect gives, from Eq. (43),

$$R_H = \left[\frac{e \tau(\varepsilon_{\text{max}})}{m^*} + \frac{\langle \Delta g(\varepsilon_{\text{max}}) \rangle \langle \sigma_z \rangle_{\varepsilon_{\text{max}}}}{B_z} \right] \frac{1}{\sigma_{xx}(\text{band})}. \quad (45)$$

In this form, we can see that the anomalous term easily competes with the normal term since $\tau \sim 10^{-14} \text{ s}$, $\langle \Delta g \rangle \sim 10^{-2} - 10^{-1}$, and $\langle \sigma_z \rangle_{\varepsilon_{\text{max}}} \sim 0.1 - 1$. Such a large effect is very difficult to achieve with hopping interference paths over third neighbors, even though the square-root dependence on resistance predicted by hopping theory is actually closer to experiment than the linear one predicted here in Eq. (45). The thermopower and resistance data by Osinniy *et al.*²⁹ also suggest band-edge conduction with the usual exponential density of state [Eq. (24)] (see Appendix B).

VI. DISCUSSION AND CONCLUSION

Our analysis suggests that hopping dominates only the very low-temperature region, and even then, it is most likely cluster-to-cluster hopping so that energy differences will depend on the cluster magnetism and not just on disorder. For temperatures above $T \sim 10 \text{ K}$, we believe that the Hall coefficient, magnetoresistance, and thermopower are due to band-edge conduction. The observed *Mott-type* laws are most likely caused by Coulomb correlations. In band-edge transport, any value of ν between 0 and 1 can be expected. The stronger the correlations, the smaller ν . Experiment does not give a unique value of ν , but it is always band-edge conduction which is observed. In band conduction, the carriers are subjected to incomparably stronger magnetoresis-

tance and Hall forces then in hopping conduction. This gives rise, in particular, to the anomalous Hall effect. In Appendix B, we show that the conductivity and thermopower data of Osinniy *et al.*²⁹ also agree with the model of delocalized band transport near the mobility edge. It appears that the temperature dependence of the transport data for insulating samples depends, to some extent, on sample preparation. When the totality of the available data in the insulating regime of GaMnAs, except those at the lowest of temperatures, is examined, it appears to be either activated transport or correlation band transport which is the dominant mechanism. There are strong indications that Mott hopping plays a secondary role at this temperature and ferromagnetic concentration range. There is, rather, a transfer process by way of which a localized hole is annihilated and created elsewhere via thermal fluctuations and is a band carrier in the intermediate state where it lives long enough to be subject to the band Hall forces and spin-disorder scattering. In other words, the long-range band transfer mechanism that gives rise to the ferromagnetism in the material is also the mechanism that determines the transport. The charge-transfer process is predominantly over the band with a Shklovskii-Efros-type²⁷ correlation band edge. The picture we propose is similar to that of Durst *et al.*⁵² for this regime. It is the dynamics of the shared spins when looked at as real or virtual band excitations, which determines the magnetotransport and Hall transport. At very low temperatures there must be a region in which hopping at the Fermi level dominates. It would be interesting to look for it experimentally. We predict that in this range the MR and Hall effect are given by Eq. (25) together with Eqs. (30) and (15) and are much smaller. Very recently, weak localization in GaMnAs has been observed.^{53,54} This regime is not exactly the one we study here (ε_f is a somewhat farther from the mobility edge in the insulating regime than in the weak localization regime) but their results are in agreement with our model.

We have presented a reasonably complete theory of transport in magnetic semiconductors of the GaMnAs type, when the Fermi level is in the region of localized states, in the presence of disorder and magnetism. The available data are, unfortunately, sparse and scattered and there does not seem to be a complete set of data on any one set of samples. The formulas presented here have a wide range of applications. The principal result of the paper is that we conclude that transport in Mn-doped GaAs is due to activated transport rather than variable range hopping. Finally, the possibility of magnetic-cluster-to-cluster hopping and tunneling should be taken seriously and investigated in detail. The resulting magnetoresistivity and Hall-effect properties of such systems are not easy to guess.

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APPENDIX A: STEEPEST-DESCENT INTEGRAL

Using a steepest-descent approach with Eq. (38) to calculate the conductivity and Hall effect and neglecting the energy dependence, apart from the density of states and Fermi function, we have an integral of the type (assume $\varepsilon_l=0$)

$$I \sim \int_{\varepsilon-\varepsilon_f}^{\infty} d\varepsilon e^{-\varepsilon/k_B T - (\varepsilon_g/\varepsilon)^\nu} G(\varepsilon). \quad (\text{A1})$$

We have changed the sign of the integral from “holes to electrons” for convenience. The maximum of the integrand, neglecting the energy dependence of G , is at

$$\varepsilon_{\max} \sim [\nu k_B T \varepsilon_g^\nu]^{1/(1+\nu)}, \quad (\text{A2})$$

provided that $\varepsilon_{\max} > |\varepsilon_c - \varepsilon_f|$. Clearly this condition and $\varepsilon_l = 0$ are not always satisfied and depend on the material and temperature. When this limit is not valid, the integral is controlled by the lower limit and the conductivity is activated, i.e.,

$$I \sim \int_{\varepsilon-\varepsilon_f}^{\infty} d\varepsilon e^{-\varepsilon/k_B T - (\varepsilon_g/\varepsilon + \varepsilon_l)^\nu} G(\varepsilon) \\ \sim k_B T G(\varepsilon_f - \varepsilon_c) e^{-[\varepsilon_g/(\varepsilon_f - \varepsilon_c) + \varepsilon_l]}. \quad (\text{A3})$$

When it is valid, the best value can be substituted back and gives a temperature dependence of the form

$$I \sim \frac{\varepsilon_{\max} G(\varepsilon_{\max})}{2} e^{-\varepsilon_{\max}/k_B T - (\varepsilon_g/\varepsilon_{\max})} \\ \sim e^{-[(\varepsilon_g/k_B T)^{\nu/(1+\nu)} + \nu^{1+\nu} + \nu^{-\nu/(1+\nu)}]}, \quad (\text{A4})$$

which is a Mott-type law but without VRH.

APPENDIX B: THERMOPOWER

The energy transported E_{xx} and the thermopower without interaction effects S are given by Movaghar and Schirmacher,²⁸

$$E_{xx} = \int_{-\infty}^{\varepsilon_c} d\varepsilon \left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \rho_{\text{edge}}(\varepsilon) D_0 \left| \frac{\varepsilon - \varepsilon_c}{\varepsilon_c} \right| (\varepsilon - \varepsilon_f) + E_{xx, \text{hop}}, \quad (\text{B1})$$

and

$$S = \frac{e E_{xx}}{\sigma_{xx} T}. \quad (\text{B2})$$

In the correlation model with $\nu=1/3$ and for low enough temperatures, given by $\varepsilon_{\max} > |\varepsilon_c - \varepsilon_f|$,

$$S = \frac{(\nu k_B T \varepsilon_g^\nu)^{1/1+\nu}}{e T}. \quad (\text{B3})$$

This yields a $T^{-1/4}$ law and a magnetic-field dependence, which is part of the Coulomb glass energy $\varepsilon_g(B_z)$ (effective Mott T_0) and not known at present.

In the exponential band-edge model [Eq. (24)], we have

$$S = \frac{\varepsilon_c(B_z) - \varepsilon_f(B_z)}{eT}. \quad (\text{B4})$$

Finally, in VRH, the thermopower depends on the structure of the density of states at the Fermi level. For a linear density of states, it is of the form^{28,55,56}

$$S \sim \frac{k_B^2 \rho(\varepsilon_f) (TT_0)^{1/2}}{e}. \quad (\text{B5})$$

The form which agrees with the experiments of Osinniy *et al.*²⁹ is the band-edge activated law Eq. (B4). The activated form is also what is reported in *a*-Si:H, where the exponential band-edge model [Eq. (24)] applies.⁴⁷ The resistance measured by Osinniy *et al.*²⁹ appears to also be activated. It seems that we have a material class with band conduction, and where the usual amorphous exponential density of states model applies. In this material type, Eq. (A2) does not apply, and the integral Eq. (A1) is cut off at $k_B T$ above the mobility edge. The transport is now controlled by the mobility edge and Eq. (B4) follows. Magnetic thermopower measurements in the insulating regime would be most useful, as they should obey Eqs. (B4) and (B5) and help us understand the nature of the transport mechanism. Pu *et al.*⁵⁷ showed that in the metallic regime, the magnetothermopower is anisotropic (depends on the orientation of M_z to the current). This is due to spin-orbit scattering. Note that the hopping thermopower, neglecting spin-flip energy transport, should also depend on the magnetic field via the magnetic-field dependence of the density of states ρ and the localization length α^{-1} with $T_0 = 24 \times 2.7 \alpha^3(B_z) / \{\pi k_B \rho[\varepsilon_f(B_z), B_z]\}$. Both spin-up and spin-down bands will contribute at the Fermi level. The localized states have a finite Hubbard U , so the full rigorous treatment is not trivial and needs focused attention as done in Ref. 58 for the finite U but B -field free case.

APPENDIX C: BAND REGION ABOVE THE MOBILITY EDGE

The mobility edge contribution to the conductivity, derived above, is due to the fact that magnetic fields and other *dephasing* effects counteract the quantum Anderson localization process. The B field moves the mobility edge up for holes (down for electrons). However, in the delocalized region, electrons (holes) are also subject to spin-disorder scattering which also causes strong magnetoresistance. It may well be that the spin-disorder band mechanism also contributes when we have the insulating scenario, despite the high

activation energy. For completeness and rigor we therefore need to also examine the transport dominated by carriers which are excited below (above) the mobility edge in the real band region. When the band region starts is not entirely clear but there is no reason why spin-disorder scattering should not start at the mobility edge as well. The band transport should therefore include the spin-disorder scattering contribution as calculated using the Kubo formula.⁷ We know that holes excited just below the mobility edge are well described by the self-consistent CPA solution^{4,13} so we can use it. The contribution from the CPA band conductivity can be written ($\varepsilon_c = \varepsilon_c(B_z, M_z)$) as

$$\sigma_{xx} = \frac{e^2}{\pi\Omega} \int_{-\infty}^{\varepsilon_c} d\varepsilon \left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) \rho(\varepsilon) D_{\text{CPA}}(\varepsilon, B_z), \quad (\text{C1})$$

where

$$\rho(\varepsilon) D_{\text{CPA}}(\varepsilon) = \frac{\hbar}{N} \sum_s \int d\varepsilon_k v_x^2 [\text{Im}\{G_{k,s}(\varepsilon)\}]^2 \delta(\varepsilon - \varepsilon_{k,s}). \quad (\text{C2})$$

Also we have

$$G_{k,s}(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_{k,s} - \Sigma_{s,\text{CPA}}(\varepsilon)} \quad (\text{C3})$$

In Figs. 1 and 2, we show the calculated curves for a parameter range which would correspond to transport at the mobility edge. The resistance results shown, multiplied by the activation factor $\exp[\frac{\varepsilon_f - \varepsilon_c}{k_B T}]$, would give us a measure of the spin-disorder band contribution to the magnetoresistance at low temperatures. We note that the extended state spin magnetoresistance is negative almost everywhere. This is a result of the fully self-consistent single-site approximation which treats both the magnetism and the transport on the same footing. The intuitive reason is that the material is more ordered when the local moments are aligned. When disordered, the holes are scattered from many types of potentials corresponding to the six possible S_z orientation of the local Mn 5/2 moment. When a magnetic field is applied, it enhances the order, and this reduces the resistance. Van Esch *et al.*⁷ showed that the resistance decreases with magnetization $M_z(B_z, T)$. We have used their formula to analytically simulate the CPA result coming from spin-disorder scattering in Eq. (33).

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